YOUR NAME: REGISTRATION #

(H) Bertrand and Russell (1/3) [10 points]

Teachers can be hard to understand sometimes. Case in point, the math teacher, Mr. Whitehead. Just this morning, he told the class, "It's not the case that if at least one student studied for the test, then every student failed the test." What does that even mean?

Well, the two new kids in the class, Bertrand and Russell, have come up with a plan to make sense of Mr. Whitehead's statements. They call it first-order logic (FOL), a way to map these confusing statements into an unambiguous representation. Bertrand says the whole system is built the idea of propositions, a statement that is either true or false. Propositions can be statements about people or things like *studied_for*(*John, test*) or *is_hard*(*test*). Propositions can also be combined to make more complex statements with the following symbols:

Symbol	Example statement	Interpretation	Explanation
¬	¬ *studied_for*(*John, test*)	John did <u>not</u> study for the test.	The statement is true if and only if John did not study for the test.
∧	*is_hard*(*test*) ∧ *is_long*(*test*)	The test is long <u>and</u> hard.	This statement is true whenever the test is long and the test is hard.
∨	*is_hard*(*test*) ∨ *is_long*(*test*)	The test is long <u>or</u> hard.	This statement is true if the test is long, or if the test is hard, or both.
⟹	*studied_for*(*John, test*) ⟹ *aced*(*John, test*)	<u>If</u> John studied for the test, <u>then</u> he aced it.	This is true if the statement on the right side of the arrow is always true whenever the statement on the left side of the arrow is true. If the statement on the left is false, then the whole statement is true by default (if John didn't study, we don't know how he did on the test).

"But," says Russell, "the most important part of first-order logic is the quantifiers." Quantifiers allow you to make general statements like Mr. Whitehead loves to do.

Symbol	Example statement	Interpretation	Explanation
∀	[$\forall_x$: *student*(*x*) ⟹ *studied_for*(*x, test*)]	Every student studied for the test.	The ∀ symbol makes a statement about every possible object (whether a student or not). It temporarily gives it the name *x* to make such a statement. We use the ⟹ symbol because we don't want to make any claims about whether non-students studied.
∃	[$\exists_x$: *student*(*x*) ∧ *aced*(*x, test*)]	There exists at least one student who aced the test.	The ∃ symbol makes the claim that there is at least one (possibly more) object in the universe, temporarily called *x*, that satisfies the statement listed.

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(H) Bertrand and Russell (2/3)

Bertrand and Russell also note that there are also a couple other things we can say about individuals (but not propositions or quantifiers). For example, if the names Jonathan and Jon both refer to the same person, we can say *Jon = Jonathan*. If we want to emphasize that John and Jon are different people, we can say *John ≠ Jon*.

H1. Translate Mr. Whitehead's statements into first-order logic by finding the proposition below that is equivalent to each statement and writing the letter of the proposition in the blank. Each statement has exactly one correct answer; not every proposition will be used.

	Everyone either passed or failed the test.
	Every student did not pass the test.
	Exactly one student passed the test.
	A student did not pass the test.
	It is not the case that if at least one student studied for the test, then every student failed the test.

A.	$[\exists_x : student(x) \land \neg\, passed(x, test)]$
B.	$[\exists_x : student(x) \land passed(x, test) \land [\forall_y : (student(y) \land passed(y, test)) \Longrightarrow x = y]]$
C.	$[\exists_x : student(x) \land passed(x, test) \land [\exists_y : student(y) \land passed(y, test) \land x = y]]$
D.	$[\forall_x : passed(x, test) \lor failed(x, test)]$
E.	$\neg\, ([\exists_x : student(x) \land studied\_for(x, test)] \Longrightarrow [\forall_x : student(x) \Longrightarrow failed(x, test)])$
F.	$[\exists_x : passed(x, test) \land failed(x, test)]$
G.	$[\forall_x : \neg\, student(x) \Longrightarrow passed(x, test)]$
H.	$[\exists_x : student(x) \land studied\_for(x, test)] \Longrightarrow \neg\, [\forall_x : student(x) \Longrightarrow failed(x, test)]$
I.	$\neg\, [\exists_x : student(x) \land \neg\, passed(x, test)]$
J.	$[\forall_x : student(x) \Longrightarrow \neg\, passed(x, test)]$

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(H) Bertrand and Russell (3/3)

H2. Translate first-order logic propositions into their equivalent English sentences by finding the statement below that is equivalent to each proposition and writing the letter of the statement in the blank. Each proposition has exactly one correct answer; not every statement will be used.

	$[\forall_x : student(x) \Longrightarrow studied\_for(x, test)] \vee [\forall_y : student(y) \Longrightarrow passed(y, test)]$
	$[\forall_x : student(x) \Longrightarrow [studied\_for(x, test) \vee passed(x, test)]]$
	$[\forall_x : (test(x) \wedge long(x)) \Longrightarrow hard(x)]$
	$[\exists_x : test(x) \wedge (long(x) \vee hard(x))]$
	$[\forall_x : test(x) \wedge \neg (long(x) \wedge hard(x)) \Longrightarrow \neg [\forall_y : student(y) \Longrightarrow failed(y, x)]]$

A.	There is a test that is long or hard.
B.	If a test is not long and not hard, then every student did not fail it.
C.	Every student studied for or passed the test.
D.	Every test that is long is also hard.
E.	Every student studied for the test or every student passed the test.
F.	If there is a test that is hard or not long, then at least one student failed it.
G.	Every test is long and hard.
H.	If a test is not both long and hard, then not every student failed it.